III. *Magnetic Survey of the West of France,* 1868. *By the Rev.* Stephen J. Perry, *S.J., F.R.A.S., F.M.S., &c. Communicated by the President.*

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The Magnetic Survey of the West of France, of which the following is a report, was undertaken by the Rev. W. Sidgreaves and myself at the request of the authorities of Stonyhurst College, who generously undertook to defray all the expenses of the expedition. A similar survey of the East of France will be made during the coming autumn, so as to complete the series of observations of the magnetic elements for the whole of France.

The instruments employed were those which have been in constant use at Stonyhurst Observatory for the determination of the monthly absolute values of the Dip, Declination, and Intensity. They consisted of a dip-circle by Barrow, No. 32, a unifilar by Jones, and Frodsham's marine chronometer, No. 3148. A beautiful transit-theodolite and an aneroid barometer were kindly placed at our disposal by the late Mr. Cooke.

The dip-circle was provided with three needles of the ordinary construction, the length of each being 3·54 inches, and the maximum breadth of Nos. 1 & 2 0·21 inch, whilst that of No. 3 was 0·32 inch. For the unifilar there were five magnets—two for declination observations, a third, No. 7, for vibrating and deflecting, and the remaining two, Nos. 9 & 10, for suspension in the deflection experiments. The same declination magnet was used throughout the whole series of observations, and the only deflected magnet employed was No. 9.

The reduced observations are given at considerable length in the following Tables, in order that the accuracy of the conclusions may be more reliable, and the results be more easily compared with those of past and future observers.

The moment of inertia of the deflecting magnet, No. 7, with its stirrup, for different degrees of temperature, and the coefficients in the corrections required for the effects of temperature and of terrestrial magnetic induction on the magnetic moment of the magnet, were determined at the Kew Observatory by the late Mr. Welsh.

The moment of inertia of the magnet, with its stirrup, using the grain and foot as the units of mass and of linear measure, is 5·27303. Its rate of increase for increase of temperature is 0·00073 for every 10° of Fahr.

The weight of the magnet, with its stirrup, is approximately 825 grains, and the length of the magnet is nearly 3·94 inches. The moment of inertia was determined, independently of the weight and dimensions, by the method of vibration, with and without a known increase of the moment of inertia.

The observations will, for clearness' sake, be arranged under the successive heads of Dip, Intensity, and Declination.

The Magnetic Dip.

The method of observing differed in nothing from that usually employed. The magnetic prime vertical was first found by four readings of the needle when vertical, and the circle then set so that the needle might oscillate in the magnetic meridian. Each end of the needle was read with the face of the circle first East and then West; next, the axis of the needle was changed on the agate planes, and the same observations again taken; and lastly, the whole was repeated after reversing the poles of the needle. Each pair of readings was repeated after lifting the axis of the needle on its Ys, and if a difference of 2′ was found, fresh sets were observed.

The results are contained in the following Table (I.).

Station.	Date.	G.M.T.	Number of readings.			Dip.			Mean.	Place of observation.
			1.	2.	3.	No. 1.	No. 2.	No. 3.		
	1868.	h m				° ′	° ′	° ′	° ′	
Paris	Aug. 12	11 26 9 43	36	...	34	65 50·2	....	65 55·2	65 52·7	Garden of Coll. Vaugirard.
Laval	,, 14	5 45 P.M. 10 28 A.M.	...	32	32	....	65 50·3	65 49·9	65 49·5	Maison St. Michel.
,, ...	,, 15	5 44 P.M.	32	...	...	65 48·3	....	....		
Brest	,, 18	11 9 0 4 3 41 P.M.	36	34	36	66 29·5	66 23·9	66 30·1	66 27·8	3 Rue d'Aiguillon.
Vannes	,, 22	2 59 P.M. 3 24 P.M. 10 49 A.M.	34	44	44	65 51·4	65 45·3	65 39·5	65 48·4	Garden of College.
Angers	,, 24	6 9 P.M. 4 9 P.M.	34	44	...	65 12·0	65 8·7	....	65 9·7	8 Rue du Faubourg St. Michel.
,, ...	,, 25	7 51 A.M.	...	...	32	....	....	65 8·5		
Poitiers	,, 26	3 8 4 9	34	...	34	64 33·9	....	64 26·5	64 29·4	Meadow of the College.
,, ...	,, 27	10 0 4 13	42	34	...	64 32·9	64 26·3	....		
Bordeaux ...	,, 29	9 3 10 3	34	32	34	63 25·6	63 22·0	63 22·6	63 24·2	Grounds of College.
,, ...	,, 30	9 3	32	...	...	63 26·6	....	....		
Abbadia......	Sept. 1	3 18	44	...	...	62 30·9	....	....	62 29·0	Magnetic Observatory.
,, ...	,, 2	....	34	36	...	62 32·7	62 23·3	....		
Bayonne......	,, 5	5 30 P.M. 4 20	32	...	30	62 30·2	....	62 32·5	62 31·4	Lamont's Station.
Pau............	,, 7	5 28 P.M. 3 25	44	...	34	62 3·5	....	61 53·4	61 59·4	
,, ...	,, 8	....	...	40	...	....	62 1·4	....		
Toulouse ...	,, 9	6 10 P.M.	34	...	...	62 4·1	....	....	62 2·2	Grounds of College.
,, ...	,, 10	9 13 A.M. 10 40	...	40	38	....	62 1·2	62 1·2		
Périgueux ...	,, 12	11 23 5 28	36	...	40	63 27·8	....	63 22·1	63 25·0	Garden of the Grand Séminaire.
Bourges	,, 14	11 3 2 58	32	...	42	64 34·6	....	64 32·7	64 33·7	Rue neuve St. Laurent.
Paris	,, 16	9 28 10 38 11 43	40	34	36	65 55·3	65 56·9	65 57·0	65 54·8	Garden of Coll. Vaugirard.
,, ...	,, 17	9 3	44	...	...	65 50·1	....	....		
Amiens	,, 20	10 35 4 38	38	...	16	66 41·3	....	66 36·5	66 41·3	Garden of St. Acheul.

In the above Table the G.M.T. is arranged in the same order as the observations. The exact site of each separate observation has been carefully noted, but the details are herein omitted for brevity's sake. The fullest inquiries have invariably been made as to the exact position of any underground iron pipes that might interfere with the observations.

In the following calculation of the most probable dip at each station, I have omitted the observations taken at Vannes with needle No. 3, since these were made during heavy rain and under a lofty but covered ball court. The readings of No. 3 at Amiens have also been excluded, since they were only taken as a verification of the results obtained with No. 1, and were only single readings.

The latitudes and longitudes in Table II. are taken from the 'Connaissance des Temps,' and the differences between the latitude and longitude of Paris, and those of the other stations, have been calculated in geographical miles by aid of the Table in LOOMIS's 'Practical Astronomy,' and each verified by measurement on the geological map of MM. DUFRÉNOY et ELIE DE BEAUMONT.

TABLE II.

Station.	Latitude.	Longitude.	Difference of Latitude.	Difference of Longitude.
	° ′ ″	h m s	geogr. miles.	geogr. miles.
Paris	48 50 11	0 0 0		
Laval	48 4 7	12 27 W.	− 53	+143
Brest	48 23 32	27 19	− 31	+318
Vannes	47 39 30	20 23	− 81	+235
Angers	47 28 17	11 34	− 94	+134
Poitiers	46 34 55	7 59	−156	+ 93
Bordeaux	44 50 19	11 40	−276	+138
Abbadia	43 23 7	16 21	−376	+196
Bayonne	43 29 29	15 16	−369	+183
Pau	43 17 44	10 51	−383	+130
Toulouse	43 36 33	3 35	−361	+ 43
Périgueux	45 11 4	6 28	−252	+ 76
Bourges	47 4 59	15 E.	−121	− 3
Amiens	49 53 43	8 W.	+ 73	+ 2

For the latitude and longitude of the Magnetic Observatory at Abbadia I am indebted to the kindness of M. D'ABBADIE, Membre de l'Institut, who rendered us every assistance during our stay in the South of France. The pillar on which most of the observations at this station were taken stands at about 800 yards N. of the Astronomical Observatory.

In forming the equations of condition, by which the most probable value of the dip at each station can be determined by the method of least squares, I have chosen Paris as the origin of coordinates for several reasons. The chief of these arises from the fact that Paris is practically the centre of France, and thus observations could easily be made there at the beginning and end of the Survey of 1868, and also of 1869. Add to this that frequent observations have in past times been made there, and that the able staff under the direction of M. LE VERRIER keep up a continued series of determinations of the magnetic elements. Lastly, the nature of the soil guarantees a perfect freedom from the disturbing influence of igneous rocks, &c.

Table III. contains the equations of condition formed from the data in Tables I., II.

$$\begin{aligned}
4{\cdot}825&=\delta+143x+53y\\
5{\cdot}497&=\delta+318x+31y\\
4{\cdot}807&=\delta+235x+81y\\
4{\cdot}162&=\delta+134x+94y\\
3{\cdot}490&=\delta+93x+156y\\
2{\cdot}403&=\delta+138x+276y\\
1{\cdot}483&=\delta+196x+376y\\
1{\cdot}523&=\delta+183x+369y\\
0{\cdot}990&=\delta+130x+383y\\
1{\cdot}037&=\delta+43x+361y\\
2{\cdot}417&=\delta+76x+252y\\
3{\cdot}562&=\delta-3x+121y\\
5{\cdot}688&=\delta+2x-73y
\end{aligned}$$

In these equations δ=the dip at the central station diminished by 61°; and $x=r\cos u$, $y=r\sin u$, where u is the angle which the isoclinal lines make with the meridian, and r is the increase in the angle of the dip for every change of a geographic mile in the direction normal to the isoclinal lines.

Solving these equations by the method of least squares, we obtain the following equations:—

$$\left\{\begin{aligned}
41{\cdot}884&=13\delta+1690x+2480y\\
5718{\cdot}258&=1690\delta+319466x+326949y\\
4912{\cdot}379&=2480\delta+326949x+757700y.
\end{aligned}\right.$$

These give

$$\left\{\begin{aligned}
487347{\cdot}924&=-59137\delta+239731870x\\
19552806{\cdot}880&=3699700\delta+469679480x;
\end{aligned}\right.$$

$$\therefore\ 44585338=9147115\delta,\quad \therefore\ \delta=4{\cdot}874.$$

Hence the most probable dip at Paris derived from the observations taken at the other stations is $65^\circ{\cdot}874=65^\circ\ 52'{\cdot}44$.

By substitution in the above equations we find the values of x, y, u, and r,

$$x=0{\cdot}0032352,\quad y=-0{\cdot}0108651,\quad u=-73^\circ\ 25'{\cdot}10,\quad r=0^\circ{\cdot}0113.$$

The isoclinal lines are therefore in the direction

N. 73° 25′ 10″ E. to S. 73° 25′ 10″ W.,

and the distance between the lines representing the difference of 30′ in the dip will be 44·25 geographical miles.

The substitution of these values of x, y, and δ in our original equations will give us the values of the computed dip at each station.

TABLE IV.

Station.	Computed Dip.	Observed Dip.	Obs. – Comp.	(Error)^2.
Paris	65° 52′·44	65° 53′·75	+1′·31	
Laval	65 46·02	65 49·5	+3·48	12·1104
Brest	66 33·96	66 27·8	–6·16	37·9456
Vannes	65 45·24	65 48·4	+3·16	9·9856
Angers	65 17·16	65 9·7	–7·46	55·6516
Poitiers	64 28·80	64 29·4	+0·60	0·3600
Bordeaux	63 19·32	63 24·2	+4·88	23·8141
Abbadia	62 25·38	62 29·0	+3·62	13·1044
Bayonne	62 27·42	62 31·4	+3·98	15·8404
Pau	62 7·98	61 59·4	–8·58	73·6164
Toulouse	62 5·46	62 2·2	–3·26	10·6276
Périgueux	63 22·92	63 25·0	+2·08	4·3264
Bourges	64 33·00	64 33·7	+0·70	0·4900
Amiens	66 40·44	66 41·3	+0·86	0·7396

$[\nu^2]=258·6121$

The most probable error for any single observation is then given by the formula $r=g.\sqrt{\frac{[\nu^2]}{m-1}}$, where $g=0·6745$, and $m=13$; ∴ Probable error $=3'·13$.

The disturbing influence from geological causes may be judged of by Table V., which is drawn up from the map of MM. DUFRÉNOY and ELIE DE BEAUMONT, Ingénieurs des Mines.

TABLE V.

Station.	Error.	Geological nature of the position.
Pau	8′·58	Terrain tertiaire supérieur.
Angers	7·46	Terrain de transition.
Brest	6·16	Terrain crystallisé, primitif; gneiss.
Bordeaux	4·88	Dépôts postérieurs aux dernières dislocations du sol. Terr. tert. sup.
Bayonne	3·98	,, ,, ,,
Abbadia	3·62	Terrain crétacé inférieur.
Laval	3·48	Terrain de transition.
Toulouse	3·26	Dépôts postérieurs.
Vannes	3·16	Terrain crystallisé, primitif.
Périgueux	2·08	Terrain crétacé inférieur.
Paris	1·31	Terrains tertiaires inférieurs.
Amiens	0·86	Dépôts postérieurs.
Bourges	0·70	Terrain Jurassique, système oolitique.
Poitiers	0·60	,, ,,

The error at Pau was most probably due to the fine dust that filled the air, the observations having been taken not far from a building in the course of erection. The amount of error at Angers and at Brest might also partly be attributed to the less favourable situation in which the instruments were used. Most of the stations were, however, well adapted to magnetic observations, being on sedimentary rocks or later deposits, and equally free from all disturbing influences.

In constructing a map of the isoclinal lines from the above values of u and r, it must be borne in mind that the assumptions of these lines being straight, of their parallelism, or of a uniform rate of increase of the dip, are only first approximations to the truth in so extensive a tract of country as that covered by the Survey. The curvature of the lines towards the north as we approach the west should be very considerable.

The secular variation of the dip has not been taken into account in the above calculations, since the time occupied in the survey was not long enough to cause any considerable change to take place during the interval. The epoch for all the observations may therefore be taken as September 1st, 1868. Table VI. contains a comparison between the above results, and those published by Dr. LAMONT in his 'Erdmagnetismus.'

TABLE VI.

Station.	Dip, Jan. 1, 1858.	Sept. 1, 1868.	Diff. of Epoch.	Diff. of Dip.	Yearly rate of decrease.	Dip on Jan. 1, 1869.
Paris	66° 26′·5	65° 53′·75	10·8	−32′·75	3′·03	65° 52′·5
Angers	65 55·9	65 9·7	„	−46·2	4·28	65 8·4
Poitiers	65 8·3	64 29·4	„	−38·9	3·60	64 28·1
Bordeaux ...	64 5·8	63 24·2	„	−41·6	3·85	63 23·0
Bayonne......	63 6·8	62 31·4	„	−35·4	3·28	62 30·2
Toulouse ...	62 46·1	62 2·2	„	−43·9	4·06	62 1·1
				Mean	3·68	

Supposing the same rate of decrease to hold for the remaining stations, where LAMONT did not observe this magnetic element, we have for the 1st of Jan. 1869 the following results:—

Laval	65° 48′·1	Pau	61° 58′·2
Brest	66 26·4	Périgueux	63 23·9
Vannes	65 47·1	Bourges	64 32·6
Abbadia	62 27·8	Amiens	66 40·3

The dip was observed on March 3rd, 1866, by M. D'ABBADIE at his magnetic observatory, and found to be 62° 39′·15, which gives 4′·22 for the annual decrease.

Referring, now, to former determinations of the secular decrease of the magnetic dip at Paris, we find, from a memoir by M. G. AIMÉ, "Sur le magnétisme terrestre"*, that the mean annual decrease between the years

1671 and 1754 was 1′·7
1754 and 1780 „ 1·0
1780 and 1806 „ 6·0.

From 1780 to 1830 the yearly diminution in the decrease amounted to 0′·051, as we learn from a Table in General SABINE'S article in the Report of the British Association for 1838.

Comparing, now, the results of the Dip Observations taken in 1858 and 1868 with the mean of those obtained by ARAGO, HUMBOLDT, and MATHIEU from 1825 to 1830, we find the annual decrease of the dip at Paris to be 2′·82 for 1843, whilst that for 1863–4 is 3′·68, which shows that there is at present a gradual acceleration in the decrease of this element amounting to about 0′·043 per annum. Dr. LAMONT gives 2′·7 as the annual diminution for 1858, which is somewhat smaller than the amount found above.

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* Published in the Annales de Chimie et de Physique, 3e série, t. xvii.

The Magnetic Intensity.

The method invariably adopted for determining the horizontal component of the earth's magnetic force was that of vibrations and deflections.

The horizontal, vertical, and total forces are calculated to English measure, one foot, one second of mean solar time, and one grain being assumed as the units of space, of time, and of mass.

The vertical and total forces are obtained from the absolute measure of the horizontal force and the dip.

The observed times of vibration are entered in the Table without correction.

The time of one vibration has been obtained from the mean of twelve determinations of the time of 100 or of 200 vibrations.

In deducing from the observed vibrations and deflections the product and ratio of the magnetic moment of the magnet and the earth's horizontal magnetic intensity, the induction and temperature corrections have always been applied, and the observed time of vibration has been corrected for the effect of torsion of the suspending thread.

The induction coefficient μ is 0·000244.

The temperature corrections have always been obtained from the formula

$$q(t_0-35^\circ)+q'(t_0-35^\circ)^2,$$

where t_0 is the observed temperature, and 35° FAHR. the adopted standard temperature. The values of the coefficients q and q' are respectively 0·0001128 and 0·000000436.

The correction for error of graduation of the deflection bar at 1 foot is +0·00004 ft., and at 1·3 foot +0·000064 ft.

It has been found necessary to apply the correction $\left(1-\frac{\mathrm{S}}{86400}\right)$ for the rate of the chronometer at two stations only, *i. e.* at Laval and at Bordeaux, where the rate was respectively $+2^{\mathrm{s}}\cdot68$ and $+2^{\mathrm{s}}\cdot18$; at the other stations it was always less than 2^{s}.

In the calculations of the ratio $\frac{m}{\mathrm{X}}$, the third and subsequent terms of the series $1+\frac{\mathrm{P}}{r^2}+\frac{\mathrm{Q}}{r^4}+\ldots$ have always been omitted.

The value of the constant P was found to be −0·002797.

The angular measure of one division of the scale in the vibration-apparatus was found to be $=2'\cdot26$.

The value of $\pi^2\mathrm{K}$ at 90° is 1·71636; this was deduced by MR. WELSH of Kew, from observations made with three inertia cylinders.

No correction has been applied for semiarcs of vibration, which were always small.

Table VII.

Station.	Date 1868.	G.M.T.	Distance of centres of magnets.	Temp.	Observed deflection.	Log $\frac{m}{X}$.	Date.	G.M.T.	Temp.	Time of one vibration.	Log mX.
		h m		°	° ′ ″			h m	°		
Paris	Aug. 11	10 48	1·0	77·8	13 30 22	9·07147	Aug. 10	6 9 P.M.	85·8	5·11042	0·30100
		11 19	1·0	80·5	13 29 43	9·07134	11	9 23	78·3	5·11138	0·30029
		11 57	1·3	79·2	6 6 27	9·07135		9 31	......	5·11294	0·30003
										5·11173	0·30023
Laval	,, 15	11 47	1·0	72·5	13 28 40	9·07016	,, 15	8 5	69·0	5·10733	0·30037
		3 21	1·0	77·9	13 27 26	9·06992	......	9 13	......	5·10745	0·30035
		3 44	1·3	78·2	6 5 35	9·07025	......	......	......	5·10745	0·30035
								9 46	70·2	5·10717	0·30048
								9 55	70·8	5·10676	0·30060
									70·5	5·10694	0·30055
Brest	,, 20	7 53	1·0	61·5	13 47 33	9·07919	,, 20	6 28 A.M.	58·8	5·15046	0·29259
		8 18	1·3	61·9	6 14 28	9·07941	......	6 36	58·8	5·15042	0·29260
										5·15044	0·29259
Vannes	,, 21	10 52	1·0	66·0	13 29 47	9·07026	,, 21	3 46	68·0	5·09825	0·30202
		11 27	1·3	65·5	6 6 4	9·06985	......	3 54	66·9	5·09760	0·30206
		11 31	1·0	65·7	13 29 10	9·06994	......	......	67·9	5·09783	0·30209
							......	6 37	62·0	5·09384	0·30237
Angers	,, 24	9 18	1·0	65·2	13 13 44	9·06168	,, 24	2 24	70·2	5·05212	0·30995
		9 49	1·0	63·8	13 13 44	9·06157					
Poitiers	,, 26	5 33	1·0	66·0	12 52 31	9·05017	,, 26	6 13	64·2	5·01525	0·31585
								6 21	63·7	5·01475	0·31586
									63·9	5·01500	0·31583
Bordeaux ...	,, 29	2 29	1·0	70·2	12 28 5	9·03671	,, 29	4 8	71·0	4·94079	0·32922
		3 12	1·3	70·5	5 38 44	9·04311	,, 30	5 22	71·2	4·94679	0·32817
								6 46	70·5	4·94417	0·32856
Abbadia	Sept. 1	5 10	1·0	73·9	12 8 1	9·02540	Sept. 1	3 54	76·8	4·87851	0·34064
		5 55	1·3	73·5	5 30 9	9·02575	......	4 20	75·5	4·87675	0·34085
								4 28	75·5	4·88079	0·34013
										4·88044	0·34019
Bayonne......	,, 5	2 43	1·0	89·5	12 5 37	9·02522	,, 5	1 35	95·0	4·88179	0·34140
								1 54	95·2	4·88275	0·34124
Pau............	,, 7	11 26	1·0	81·5	12 1 14	9·02199	,, 7	10 2	79·0	4·86188	0·34379
								10 10	79·0	4·86113	0·34392
										4·86150	0·34386
Toulouse ...	,, 9	2 10	1·0	84·8	12 2 7	9·02277	,, 9	11 5	80·9	4·85392	0·34534
		2 40	1·3	85·9	5 27 1	9·02260	......	11 28	82·2	4·85368	0·34547
							,, 10	2 52	87·6	4·85375	0·34587
								3 16	87·2	4·85504	0·34560
								3 24	87·5	4·85396	0·34583
								3 28	87·5	4·85283	0·34603
Périgueux ...	,, 12	3 48	1·0	80·7	12 29 1	9·03809	,, 12	2 35	80·8	4·94071	0·32995
Bourges	......	......	......	......	......	......	,, 14	4 30	70·0	5·01896	0·31554
Paris	,, 16	5 58	1·0	66·7	13 29 38	9·07024	,, 16	2 34	72·4	5·12921	0·29683
							,, 18	8 17 A.M.	62·2	5·11971	0·29775
Amiens	,, 20	2 58	1·0	70·5	13 47 11	9·07967	,, 20	8 36 A.M.	60·2	5·18455	0·28667
		3 16	1·3	69·1	6 14 28	9·07994	......	8 40	60·4	5·18425	0·28673
								8 44	60·9	5·18392	0·28683

The values of the total force and of its components, as also the changes in the magnetic moment of the deflecting magnet, are at once deduced from the above Table.

TABLE VIII.

Station.	Horizontal Force.	Vertical Force.	Total Force.	Moment of Magnet.
Paris	4·1162	9·1926	10·0721	0·4852
Laval	4·1226	9·1840	10·0668	0·4845
Brest	4·0424	9·2807	10·1228	0·4852
Vannes	4·1310	9·1948	10·0802	0·4854
Angers	4·2088	9·0926	10·0196	0·4851
Poitiers	4·2938	8·9979	9·9701	0·4820
Bordeaux ...	4·4093	8·8064	9·8487	0·4834
Abbadia	4·5439	8·7225	9·8351	0·4820
Bayonne......	4·5504	8·7498	9·8626	0·4823
Pau............	4·5807	8·6113	9·7539	0·4819
Toulouse ...	4·5867	8·6397	9·7818	0·4833
Périgueux ...	4·4252	8·8432	9·8887	0·4831
Bourges	4·2830	9·0045	9·9713	0·4828
Paris...	4·1070	9·1869	10·0631	0·4828
Amiens	4·0129	9·3128	10·1403	0·4823

Deflections not having been observed at Bourges, it was necessary to compute the value of m, in order to eliminate it from the product mX obtained by the vibrations. The mean value of m during the survey was 0·48335 at epoch Sept. 1st, 1868; the mean for the remainder of 1868 was 0·48037, therefore the variation per month $=\frac{298}{2\cdot5}=119\cdot2$, $\therefore$ the value on Sept. 14 should have been 0·48283. This agrees well with the mean value of m at Périgueux on the 12th and Paris on the 16th, *i. e.* 0·48294. Forming, now, our equations of condition from the above data, and solving them by the method of least squares, precisely in the same manner as in the case of the dip, we obtain the equations

$$12\cdot3419=13f+1690x+2480y, \text{ \&c.},$$

where $f=$ the total force diminished by 9.

From these we obtain

$$6279\cdot2889=-59137f+239731870x,$$
$$4089465\cdot7980=3699700f+469679480x;$$
$$\therefore\ 977426255=914711558f \text{ and } f=1\cdot06857.$$

This gives for the most probable computed value of the total force at Paris 10·0686, whereas the mean of the direct observations was 10·0676; difference =0·0010.

By substitution we at once get the quantities

$$x=0\cdot00028979,\ y=-0\cdot0008223;\ \therefore\ r=0\cdot000872, \text{ and } u=-70^\circ\ 34'\ 13''\cdot1.$$

Hence the direction of the isodynamic lines is

N. 70° 34′ 13″·1 E. to S. 70° 34′ 13″·1 W.,

and the total force changes 0·1 for every 115 geographical miles along the normal to these lines.

We can now draw up a Table of the computed errors in the observations.

TABLE IX.

Station.	Computed T. F.	Observed T. F.	Obs. – Comp.	(Error)^2.
Paris	10·0686	10·0676	–0·0010	
Laval	10·0670	10·0668	–0·0002	0·00000004
Brest	10·1352	10·1228	–0·0124	0·00015376
Vannes	10·0706	10·0802	+0·0096	0·00009216
Angers	10·0301	10·0196	–0·0105	0·00011025
Poitiers	9·9672	9·9701	+0·0029	0·00000841
Bordeaux	9·8816	9·8487	–0·0329	0·00108241
Abbadia	9·8162	9·8351	+0·0189	0·00035721
Bayonne............	9·8172	9·8626	+0·0454	0·00206116
Pau..................	9·7913	9·7539	–0·0374	0·00139876
Toulouse	9·7842	9·7818	–0·0024	0·00000576
Périgueux	9·8834	9·8887	+0·0053	0·00002809
Bourges	9·9682	9·9713	+0·0031	0·00000961
Amiens	10·1292	10·1403	+0·0111	0·00012321

$[\nu^2]=0·00543083$

The most probable error of any one observation will therefore be

$$=r=0·6745\sqrt{\frac{[\nu^2]}{12}}=0·0144.$$

For the sake of comparison with the results of former observations, and in particular of those of Dr. LAMONT, we will determine the direction of the lines of equal intensity for the horizontal component of the earth's magnetism.

By a process identical with that employed for the Total Force, we now find that at our central station the Horizontal Force$=4·1156$,

$$x=-0·00037605,\ y=0·0013401, \text{ and } \therefore\ r=0·0013919, \text{ and } u=-74°\ 19'\ 30''·4;$$

or the direction sought is

N. 74° 19′ 30″·4 E. to S. 74° 19′ 30″·4 W.;

the probable error in any single observation being now 0·0067.

We will next place side by side the values of the Horizontal Force for 1858 and 1868.

TABLE X.

Station.	Jan. 1st, 1858.	Sept. 1st, 1868.	Difference.
Amiens	3·9639	4·0129	+0·0490
Angers	4·1450	4·2088	+0·0638
Bayonne..................	4·4875	4·5504	+0·0628
Bordeaux	4·3730	4·4093	+0·0363
Paris	4·0685	4·1116	+0·0431
Périgueux	4·3699	4·4252	+0·0553
Poitiers	4·2341	4·2938	+0·0597
Toulouse	4·5243	4·5867	+0·0624

Hence the yearly increase of the Horizontal Force in the West of France $=0·0050$.

Dr. LAMONT gives 0·0048 as the value for 1858, therefore we may conclude that there is a slight acceleration of about 0·00002 per annum in the increase of this element.

The Magnetic Declination.

The method adopted in the determination of this element was the following.

The azimuth of a fixed horizontal mark, situated at a considerable distance, was first read with COOKE'S transit theodolite, and then a transit of both limbs of the sun taken with the same instrument, the time being noted by FRODSHAM'S chronometer. The azimuth circle having been read, the theodolite was removed, and JONES'S unifilar placed on the same tripod-stand. The reading of the fixed mark and of the collimator magnet completed the observation. The torsion of the silk thread was removed entirely, as far as that was possible, before each observation.

Brest was the only station at which the above method was departed from, and there, on account of the confined space in which the observations were taken, a distant mark could not be sighted. It was therefore considered that more accurate results would be obtained by observing the sun's transit by reflection from the mirror of the unifilar, according to Dr. LLOYD'S method. The chief reason for not using this method at other stations was that the line of collimation of the telescope was not perpendicular to the axis of the mirror, and the motion in azimuth of the axis was not sufficient to correct this error.

The correction to be applied to the azimuth reading of the sun on account of this position of the mirror is given by the formula $x=\frac{2m\sin^2\frac{\alpha}{2}}{\cos\alpha}$, where the constant $2m=\frac{\cos\alpha'\cos\alpha''}{\sin\frac{\alpha'+\alpha''}{2}\,\sin\frac{\alpha'-\alpha''}{2}}$, α, α', α'' being the altitudes of the sun at the times of observation.

The formula $\cos a=\frac{\sin h\ \cos\delta}{\sin \mathrm{A}}$ was used for determining the altitudes.

The determination of the meridian line, or of the azimuth of the sun, at each observation depends mainly for its correctness upon the knowledge of the error and rate of the chronometer employed. No pains were therefore spared to prevent all jolting of the instrument during the voyage, and no opportunity was missed of comparing it with other chronometers. The following Table is the result of these comparisons, made always with the greatest courtesy by those in charge at the several observatories and dépôts de chronomètres.

The chronometer is a large-size marine instrument by FRODSHAM, No. 3148; it has been in constant use at Stonyhurst Observatory since the beginning of 1863, and its daily rate is found by the observation of clock stars on every favourable night. Its mean daily rate previous to the journey was $0^{s}\cdot57$, and afterwards it increased to $0^{s}\cdot61$. The rate is found to vary somewhat during the year. Mr. FRODSHAM kindly examined the instrument before it was taken to France and immediately after its return, and declared it to be in perfect order.

TABLE XI.

Station.	Date.	G. M. T.	Error.	Daily rate.
		h m	m s	s
Stonyhurst Observatory	July 26	......	+0 59·14	+0·57
London, Frodsham	Aug. 7	......	+1 13·0	+1·8
Paris Observatory	„ 11	3 30	+1 15·63	+0·66
Brest Observatory	„ 17	2 15	+1 31·71	
	„ 18	2 15	+1 32·88	+1·17
	„ 19	2 15	+1 34·05	+1·17
Bordeaux, Dépôts des Chronomètres ...	„ 29	1 13	+1 29·0	
Abbadia Observatory	Sept. 2	......	+1 37·7	+2·18
Toulouse Observatory	„ 9	......	+	small
Paris Observatory..........................	„ 17	......	+1 51·5	+0·92
London, Frodsham	„ 24	......	+0 28·0	+1·0
Stonyhurst Observatory	„ 30	......	+0 34·35	
	March 14	......	+2 15·58	+0·61

At Toulouse the chronometer was only compared as to rate by the Director of the Observatory, the absolute times of comparison not being registered.

Altitudes of the sun were taken at nearly all the stations with COOKE'S small altazimuth, but these were only used as a check on the above results and for intermediate stations. These direct determinations of the rate of our instrument were exceedingly useful, from showing us that the two principal disturbances probably do not interfere in the least with the results obtained, since the first took place between Poitiers and Bordeaux, and the second between Amiens and London.

In the following Table of observations the first line of azimuths at each station was read on the theodolite circle, and the second line on the unifilar. The centre division of the magnet was always made to coincide with the centre line of the telescope, except at Vannes, where it was 0·1 div. to the apparent left. The zero of the scale of the collimator magnet was +7·2 div., each scale-division being $=2' \, 9''\cdot805$.

At Abbadia the azimuth of the mark A, *i. e.* the Biarritz Lighthouse, was accurately determined by M. ANTOINE D'ABBADIE, Président de la Société Météorologique de France.

TABLE XII.

Station.	Date.	Chronometer.	Error at Noon.	Daily Rate.	Azimuth of Sun.	Azimuth of Mark.	Azimuth of Magnet.
		h m s	m s	s			
Paris	Aug. 12	5 25 59·375	+1 16·19	+0·66	261° 5′ 10″	187° 45′ 20″	
		6 15 0	....	....	....	240 37 25	201° 40′ 25″
					"	108 45 20	
						161 37 55	"
Laval	" 14	2 26 55·75	+1 23·67	+2·68	110 12 12·5	144 12 10	
		3 10	....	....	....	257 23 15	152 4 45
Brest	" 20	10 3 17·75	+1 35·22	+1·17	252 6 12·5		
		10 26	....	....	....	....	100 57 40
Vannes	" 21	4 59 49·5	+1 36·39	+1·17	107 46 55	142 21 25	
		5 45	....	....	....	106 3 50	147 12 0
	" 22	5 34 24·08	+1 37·56	+1·17	215 55 13·3	244 26 30	
		6 38	....	....	....	101 21 10	142 20 0
					"	244 31 20	
						101 26 50	"
Angers	" 24	5 8 21·5	+1 39·90	+1·17	254 37 40	252 55 45	
		5 52	....	....	....	127 49 15	203 16 25
					"	198 30 25	
						253 24 5	"
Poitiers	" 26	10 10 5·08	+1 42·24	+1·17	155 27 18·3	115 23 10	
		11 11	....	....	....	202 38 25	266 16 45
Bordeaux ...	" 29	5 35 47·56	+1 29·0	+2·18	228 17 43·8	139 51 30	
						92 12 40	250 38 30
Abbadia......	Sept. 1	3 5	+1 35·52	+0·92	....	217 21 15	146 5 40
Bayonne......	" 5	11 18 16·13	+1 40·46	+0·92	223 25 15	135 55 55	
		12 50	....	....	....	127 15 15	215 23 45
Pau............	" 7	8 24 41·88	+1 42·30	+0·92	246 29 21·7	193 19 20	
		8 48	....	....	....	93 47 35	196 50 50
Toulouse ...	" 9	9 37 11·18	+1 44·14	+0·92	264 15 45	176 7 25	
		10 12	....	....	....	194 9 0	132 39 35
					"	220 37 25	
						238 38 20	"
Périgueux ...	" 12	9 9 42·18	+1 46·90	+0·92	127 41 58·3	250 53 57·5	
		10 4	....	....	....	206 46 45	119 21 25
Bourges	" 14	8 49 22·38	+1 48·74	+0·92	261 31 15	118 15 45	
		9 45	....	....	....	162 43 15	164 7 35
Paris	" 16	3 30 32·76	+1 50·58	+0·92	139 23 3·3	165 54 15	
		4 35 30	....	....	....	151 51 20	222 36 40
Amiens	" 20	1 32 38·75	+1 54·26	+0·92	118 42 5	234 19 0	
		2 12	....	....	....	100 14 55	112 19 35

The azimuth of the sun at the time of observation, calculated by the formula $\cot A = \cot h \, \frac{\cos (mP + \lambda)}{\sin mP}$, where $mP = \tan^{-1} (\cos h \cot \delta)$, gives the results in Table XIII.

TABLE XIII.

Station.	Azimuth of the Sun.	Declination corrected for scale-reading.
Paris	94° 13′ 13″·9	17° 48′ 1″·5
"	....	17 48 31·5
Laval	51 55 12·3	19 7 45·6
Brest	50 39 10·9	21 3 32·6
Vannes	84 6 11·4	19 55 47·0
"	89 36 59·1	20 37 19·6
"	....	20 38 9·6
Angers	86 50 31·4	19 8 39·0
"	....	19 8 49·0
Poitiers	42 11 13·4	18 21 27·1
Bordeaux	91 29 15·4	18 15 33·8
Abbadia	....	18 17 6·4
Bayonne........................	19 21 5·4	18 26 20·8
Pau..............................	68 1 9·2	17 52 21·3
Toulouse	47 48 10·3	17 10 0·7
Périgueux	48 36 28·1	17 43 34·3
Bourges	55 27 8·7	17 2 44·1
Paris	64 29 14·6	17 58 39·1
Amiens	33 41 22·6	18 21 27·8

An application of the method of least squares, similar in all details to that made use of above, furnishes the equations

$$20·125=13D+1690x+2480y, \text{ \&c.}$$

Hence

$$D=0·92921, \quad x=0·01056199, \quad y=-0·0039534;$$

$\therefore$ $r=0°·011278$ per geographical mile, *i. e.* the rate of change in the declination along the normal to the isogonic lines is 1° for every 88·7 geographical miles, and $u=-20°31'16''$, or the direction of the isogonic lines is

N. 20° 31′ 16″ E. to S. 20° 31′ 16″ W.

We might naturally expect that observations of this magnetic element would lead to much less satisfactory results than those of the Dip or Total Force; for the almost continuous, and often energetic, action of disturbing forces render any limited number of absolute readings of the declination at different stations a very uncertain guide in calculating the exact direction taken by the isogonic lines; whereas in the case of the vertical and horizontal components of the total force the daily disturbances are of a much less exaggerated character, and the maximum perturbations of not frequent occurrence. The subjoined Table will show the probable amount of error at each station.

TABLE XIV.

Station.	Computed Declination.	Observed Declination.	Diff.
Paris	17°·929	17°·891	− 2′·28
Laval	19·251	19·129	− 7·32
Brest	21·165	21·059	− 6·36
Vannes	20·091	20·279	+11·28
Angers	18·973	19·146	+10·38
Poitiers	18·295	18·358	+ 3·78
Bordeaux	18·296	18·259	− 2·22
Abbadia	18·513	18·285	−13·68
Bayonne	18·403	18·439	+ 2·16
Pau	17·788	17·873	+ 5·10
Toulouse	17·044	17·167	+12·66
Périgueux	17·736	17·726	− 0·60
Bourges	17·419	17·047	−22·32
Amiens	18·239	18·358	+ 7·14

The probable error of any single observation will therefore be 6′·9549. We are now able to determine the secular variation of the declination.

TABLE XV.

Station.	Epoch January 1st, 1858.	September 1st, 1868.	Variation.
Amiens	19° 56′ 18″	18° 21′ 27″·8	−1° 34′ 50″·2
Angers	20 16 18	19 8 44·0	−1 7 34·0
Bayonne	19 57 48	18 26 20·8	−1 31 27·2
Bordeaux	20 0 12	18 15 33·8	−1 44 38·2
Paris	19 36 18	17 53 27·8	−1 42 50·2
Périgueux	19 26 30	17 43 34·3	−1 42 55·7
Poitiers	19 56 24	18 21 27·1	−1 34 56·9
Toulouse	18 45 0	17 10 0·7	−1 34 59·3

At Angers an iron pipe was lying N. and S. at a distance of about 11 yards from the place of observation. Neglecting, therefore, this station, we find the yearly decrease of the Declination in the West of France to be 9′ 4″·9.

M. G. Aimé, in the "Mémoire" already quoted, gives the mean annual variation of the Declination at Paris for past years.

Epoch.	Declination.	Yearly variation.
1663 to 1767	0° 0′ to 19° 16′	+11′·0
1767 to 1785	19 16 to 22 0	+ 9·0
1785 to 1805	22 0 to 22 5	+ 0·2
1805 to 1817	22 5 to 22 19	+ 1·1
1817 to 1825	22 19 to 22 22	+ 0·3

Continuing this summary a step further we have

1825 to 1858, 22° 22′ to 19° 36′, yearly diminution 5′·0,
1858·0 to 1868·7, 19° 36′ to 17° 53′, yearly diminution 9′·6,

whence it appears that the declination is rapidly on the decrease, with a mean yearly acceleration in the decrease of 0′·22.

The value of the yearly diminution, as given by Dr. Lamont, is 7′·6 for 1858, which shows a steadiness in the variation of this magnetic element.

A glance at the maps which accompany this report will show at once the changes that ten years have produced in the position of the lines of equal declination, dip, and intensity. The distance between the lines remains in all cases almost constant for the same element, and the amount is moreover identical for the isoclinal and isogonic lines, the values of r being respectively 0·678 and 0·677. The angle moved through by the lines is more considerable in the cases of the isoclinal lines than for the others, and the direction of this motion is away from the astronomical meridian in the case of the isogonics and isoclinals, and scarcely perceptibly towards the astronomical meridian for the lines of equal horizontal force.

In the maps of Lamont's 'Erdmagnetismus' the lines are curved, and the epochs for the dip, horizontal force, and declination are severally August 1848, June 1848, and March 1854; whereas I have taken January 1st, 1858, as the common epoch, and drawn the lines straight, for the sake of comparison with the lines for 1868, which are laid down without any modification from the calculated values of u and r. The dotted lines, which belong to the survey of 1858, can therefore only be looked on as first approximations to the results derived from Lamont's observations. The broken lines are those obtained by the above calculations.

The numbers marked at each station are the means of the observed values, and they serve to show the degree of approximative correctness attained by the adopted method of reduction. A comparison of these maps with those of the latest surveys of the British Islands manifests a striking resemblance between the isoclinal lines in France and England. For in the Report by General SABINE for the British Association 1861, we find that the direction of the isoclinals in England was −65° 5′ in 1837, and −71° 22′ in 1860; therefore, supposing the rate of change to remain constant, we have −73° 33′·2 as the value for 1868, −73° 25′·2 being the amount found for France in August and September of the same year. The values of *r*, 0·678 and 0·644, show, however, that the lines are closer packed in France than in England, and this is confirmed by the observations taken with needle No. 1 at Kew and at Stonyhurst immediately after our return from France; for if the angles be computed on the assumption of the same data holding in England as in France, we shall have:

	Observed.	Computed.	Difference.
	°	°	′
Kew	68·087	68·241	− 9·24
Stonyhurst	69·707	70·306	−35·94

The isogonics of France are nearly in the same direction as those of Scotland, as seen in Dr. STEWART'S Report of Mr. WELSH'S Survey; but they are nearly twice as close in Scotland as in France, the values of *r* being 0′·677 in 1868 for the latter country, and 1′·465 for the former in 1858.

The greatest difference is in the isodynamics, their angle for England being −58° 32′·7, and *r*=0·00106, whilst for France at the same epoch, 1868, the values are −70° 34′ 13″·1 and 0·00087.

The following is a complete Table of all the elements at the Epoch Jan. 1st, 1869.

TABLE XVI.

Station.	Dip.	Declination.	Horizontal Force.
	° ′	° ′	
Abbadia	62 27·8	18 14·10	4·5456
Amiens	66 40·3	18 18·96	4·0143
Angers	65 8·4	19 5·58	4·2106
Bayonne	62 30·2	18 23·46	4·5520
Bordeaux	63 23·0	18 12·54	4·4110
Bourges	64 32·6	17 0·18	4·2845
Brest	66 26·4	21 0·30	4·0442
Laval	65 48·1	19 4·38	4·1245
Paris	65 52·5	17 50·46	4·1133
Pau	61 58·2	17 49·50	4·5823
Périgueux	63 23·9	17 40·92	4·4268
Poitiers	64 28·1	18 18·36	4·2955
Toulouse	62 1·1	17 7·32	4·5883
Vannes	65 47·1	20 13·50	4·1328
Annual Variation	−3·68	−9·1	+0·0050
Acceleration	0·043	0·19	0·00002

In none of the previous reductions have I ventured to introduce a correction for any magnetic disturbance that might have existed at the time of observation, or for diurnal range, since there were no self-recording magnetographs in France by which these corrections could be accurately determined, and any correction founded on the supposition of the simultaneous similar action of the disturbing forces in England and in France might appear somewhat arbitrary. That a correction, however, might be applied with advantage is rendered more than probable by the results of the comparison of the Kew and Lisbon magnetograms, and by the great similarity between the daily range in England and in Italy which I remarked in some of the Florence declination-curves sent by Signor DONATI. It would seem, from the comparisons made by Dr. STEWART and Senhor CAPELLO, that the declination and horizontal-force disturbances at Kew and Lisbon are simultaneous, in the same direction, and in the proportion of 1·6 to 1 for the declination, and 1·8 to 1 for the horizontal force; whilst, on the other hand, Dr. STEWART remarks that there is "very little likeness between the vertical-force curves."

We will therefore assume that the perturbations of the declination and horizontal force are simultaneous in England and in France, and in the proportion of 1·3 to 1 and 1·4 to 1 respectively, and we will take the corrections from the magnetograms obtained at Stonyhurst Observatory during the survey, since the Kew and Stonyhurst curves may be considered as almost identical.

From hourly measurements of the undisturbed portions of our declination-curves from Dec. 16th, 1868 to Jan. 16th, 1869, we obtain 2·147 as the mean reading of the ordinate for Jan. 1st, 1869; and taking the difference between this and the ordinate at the time of each observation, we obtain a number which, when divided by 1·3 and multiplied into 28′ 38″·875, the coefficient of the declination magnetograms, gives us the correction in arc for reducing the French observations to their mean value on Jan. 1st, 1869. No correction for torsion of thread has been found necessary.

The results obtained from the corrected observations compared with the values found above, will best show what is gained by this correction.

	From uncorrected observations.	From corrected observations.
$x=$	0·010562	0·010385
$y=$	−0·003953	−0·003833
$u=$	−20° 31′ 16″	−20° 15′ 35″
$r=$	0·011278	0·01107
Error of Paris	−2′·28	1′·26
Probable error of any observation	6′·95	7·15.

These figures of themselves would scarcely justify us in giving much weight to this correction.

The horizontal-force magnetogram seems to offer a surer means of improving our results. The standard ordinate has been obtained from hourly measurements of the curves on undisturbed days in Aug. and Sept. 1868, the gradual loss of magnetism of the suspended magnet being taken into account. The coefficient of the curve is 0·02689. In this case the amount of difference between the mean observed and the computed values at each station is generally diminished by the correction; but, owing to the smallness of these differences and to the trace of the magnetograph having been lost at the time of the observations at Angers and Bordeaux, where the differences were greatest, the total change of the values is inconsiderable.

The following comparison will give an idea of the results:—

	From uncorrected observations.	From corrected observations.
$x=$	$-0·00037605$	$-0·00037562$
$y=$	0·0013401	0·0013398
$u=$	74° 19′ 30″·4	74° 20′ 19″·5
$r=$	0·0013919	0·0013914
Error at Paris	0·0040	0·0034
Mean probable error . . .	0·0067	0·0065

A similar correction might be applied to the Total Force and the Dip; but since the simultaneous variations of the vertical force are less understood than those of the horizontal force or the declination, the attempt could scarcely be expected to lead to any satisfactory result in the present state of our knowledge.

APPENDIX.

The observations made with the same instruments at Loyola on Sept. 3 and 4 are not taken into account in the above reductions, since we wished to confine ourselves entirely to the West of France. It may, however, be of some use to subjoin a synopsis of the results obtained.

Sept. 3rd, 3^h 11^m $9^s·26$ G.M.T. Azimuth of sun 165° 35′ 46″·7. Azimuth of mark A 141° 53′ 40″, read with the altazimuth.

3^h 43^m Azimuth of magnet 229° 49′ 5″. Azimuth of mark A 206° 42′ 40″, read with the unifilar.

Correction for zero-point of scale of collimator magnet $-15′$ 34″·6.

The declination is not deduced from want of sufficiently reliable data as to latitude and longitude.

Sept. 3rd, 6^h 26^m, temp. 71°·6 Fahr. Observed deflection 12° 7′ 2″. Distance between centres of magnets 1 ft.

„ 4^h 54^m, „ 77°·0 Fahr. Observed time of one vibration $4^s·87359$.

Hence

Moment of magnet . . . . . . =0·48205
Horizontal force . . . . . . =4·5546
Vertical force . . . . =8·7245
Total force . . . . =9·8419

Sept. 4. No. 1 Needle. Observed dip 62° 26′.

Magnetic Dip.

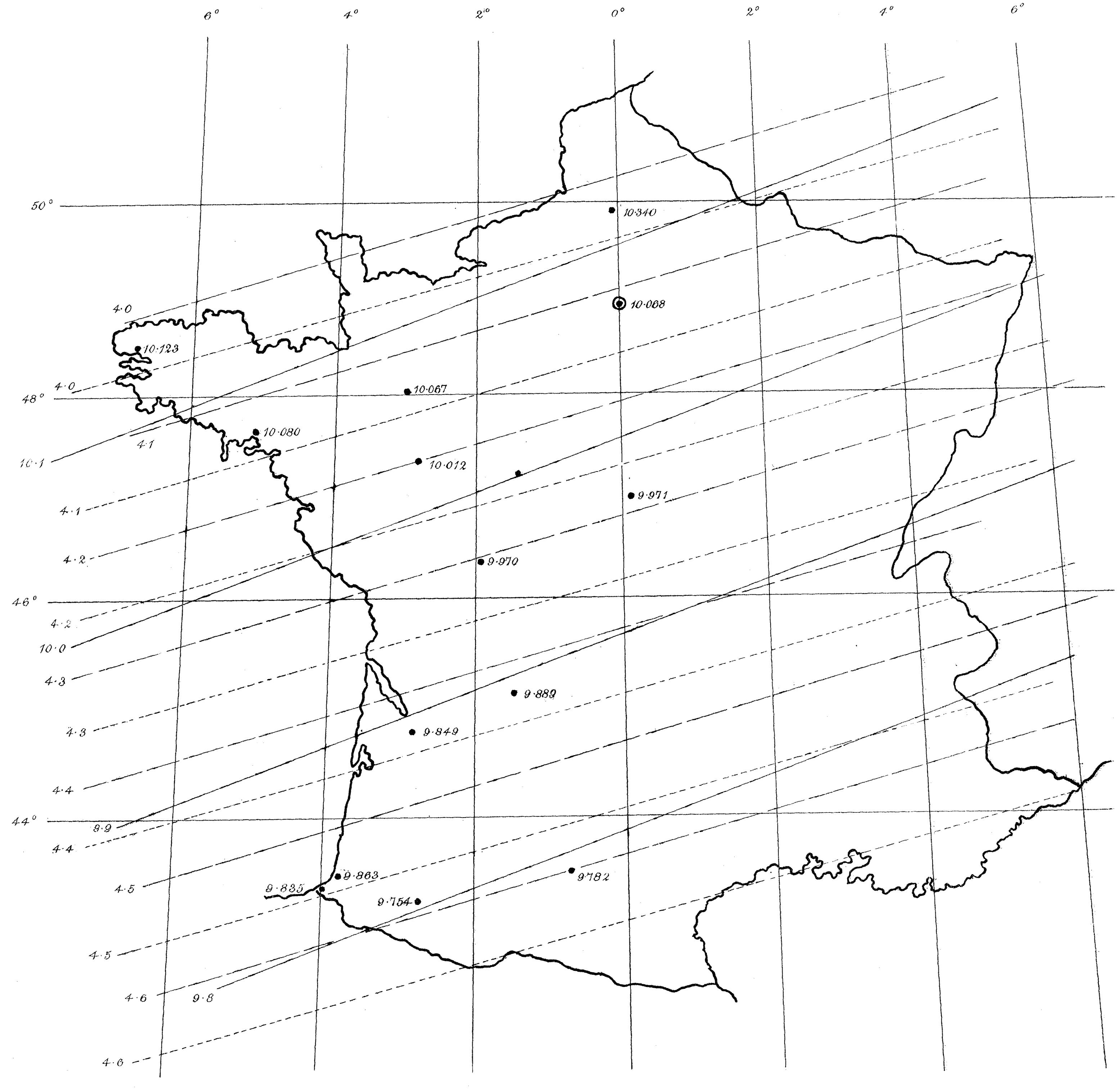
Isoclinals, Epoch 1858 ....

" " 1868 — — — — —

W.H.Wesley. lith.

W.West imp.

Magnetic Intensity.



Isodynamics, Epoch 1868 ————————

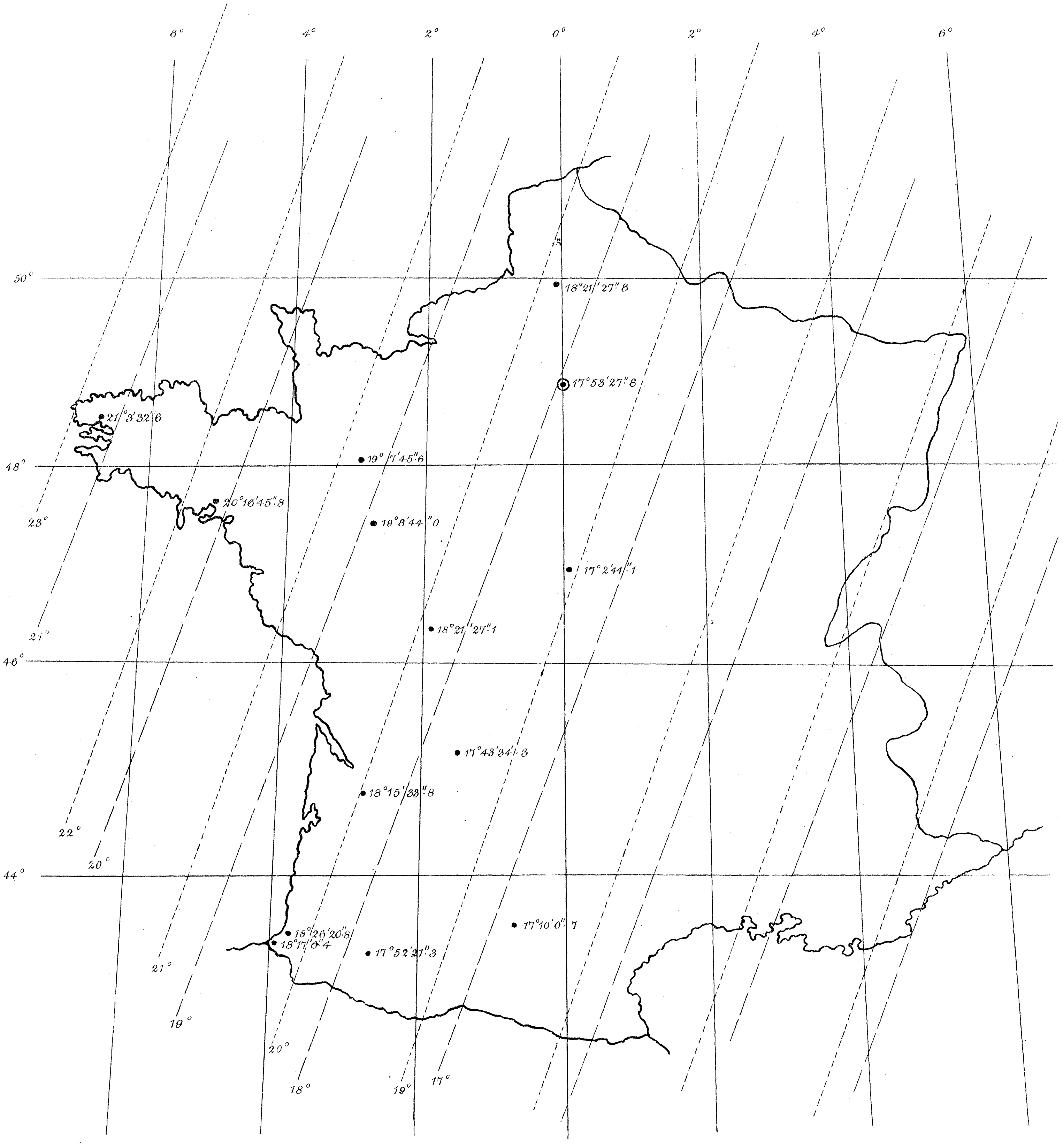
Lines of equal Horizontal Force, „ 1858 - - - - - - - - - -

„ „ „ „ „ „ 1868 — — — — — —

W.H. Wesley, lith.

W West imp.

Magnetic Declination



Isogonics Epoch 1858 - - - - - - - - -
1868 — — — — —

W H Wesley lith.
W. West imp.